## Prof. Dr.-Ing.habil.Dr.h.c. Holm Altenbach

#### Some personal information

Chair of Engineering Mechanics, Institute of Mechanics, Faculty of Mechanical Engineering,

R)

Graduate School

Micro-Macro-Interactions of Structured Media and Particle Systems,

Otto-von-Guericke-Universität Magdeburg (Germany)







### **Personal Data**

### Short CV (I)

- May 8<sup>th</sup>, 1956 born in Leipzig parents: Martina & Prof. Dr.-Ing.habil.Dr.h.c. Johannes Altenbach
- since 1957 living in Magdeburg
- 1962 1974 school education in Magdeburg & Halle
- 1974 1980 student at the Leningrad (St. Petersburg) Polytechnic Institute
- 1980 1995 Otto-von-Guericke-University Magdeburg (Assistant, Senior Assistant)
- 1983 PhD Leningrad (St. Petersburg) Polytechnic Institute
- 1984 Facultas docendi (TH Magdeburg)
- 1987 DSc Leningrad (St. Petersburg) Polytechnic Institute

### **Personal Data**

#### Short CV (II)

- 1991, 1992, 1993, 1995, 1996, 2011 visiting scientist/professor in Kharkiv (Ukraine), Bochum, Bratislava (Slovakia), Riga (Latvia), Nagoya
- 1993 Privatdocent in Magdeburg
- 1995 appointment Professor of Structural Mechanics in Lausanne (Switzerland)
- 1995 apl. Professor in Magdeburg
- 1996 appointment Professor of Engineering Mechanics in Halle
- 1998 2000 Vice Dean
- 2000 2011 Dean
- 2008 Dr.honoris causa NTU Kharkiv Polytechnic Institute (Ukraine)
- 2011 Professor of Engineering Mechanics in Magdeburg

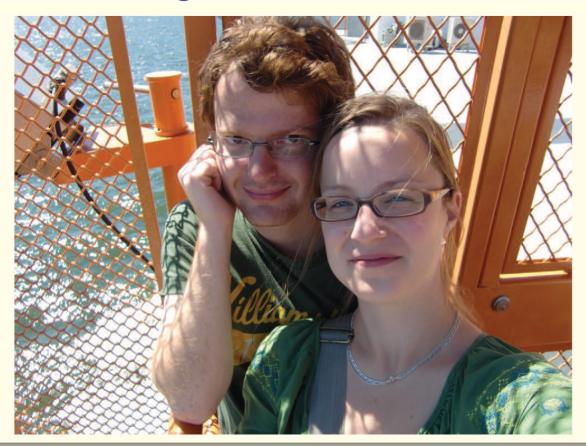
## My wife Natalija (Natascha)



## My daughter Nadina with her younger daughter



## My son Fabian with his girlfriend



## My granddaughters Sarah & Anna



# **Chair of Engineering Mechanics**

My international team - last Christmas



# Magdeburg

## My hometown



#### **Journals**

- 2004 to date Managing Editor of the Journal of Applied Mathematics and Mechanics (ZAMM, Wiley)
- 2005 to date Editor-in-Chief of the Journal of Applied Mathematics and Mechanics
- Board member:
  - Mechanics of Composite Mechanics (Riga, Latvia),
  - Maintenance and Reliability (Lublin, Poland),
  - Technische Mechanik (Magdeburg),
  - Journal of Strain Analysis and Engineering Design (UK, Saga)
  - Mechanics of Advanced Materials and Structures Journal (UK, Taylor & Francis)
- Advisory Editor Continuum Mechanics and Thermodynamics (Springer)

### Co-editor (I)

- Springer Series Advanced Structured Materials
- CISM
  - Altenbach, H.; Skrzypek, J.J. (Eds.): Creep and Damage in Materials and Structures. CISM Courses and Lectures No. 399. Wien, New York: Springer-Verlag, 1999
  - Altenbach, H.; Becker, W. (Eds.): Modern Trends in Composite Laminates Mechanics. CISM Courses and Lectures No. 448. Wien, New York: Springer-Verlag, 2003
  - Altenbach, H.; Öchsner, A. (Eds.): Cellular and Porous Materials: Modeling - Testing - Application. CISM Courses and Lectures No. 521.
     Wien, New York: Springer-Verlag, 2010
  - Altenbach, H.; Eremeyev, V.A. (Eds.): Generalized Continua from the Theory to Engineering Applications (under preparation)
- Co-editor Encyclopedia of Continuum Mechanics (Springer, under preparation)

### Co-editor (II)

- Kienzler, R.; Altenbach, H.; Ott, I. (Eds.): Critical Review of the Theories of Plates and Shells, New Applicatitions. Berlin: Springer-Verlag, 2004 (Lecture Notes in Applied Computational Mechanics 16)
- Öchsner, A.; daSilva, L.F.M.; Altenbach, H. (Eds.): Materials with Complex Behaviour - Modelling, Simulation, Testing, and Applications. Advanced Structured Materials, Vol. 3 - Berlin: Springer-Verlag, 2010
- Altenbach, H.; Erofeev, V.; Maugin, G.A. (Eds.): Mechanics of Generalized Continua. Advanced Structured Materials, Vol. 7 -Berlin: Springer-Verlag, 2011
- Altenbach, H.; Eremeyev, V.A. (Eds.): Shell-like Structures Nonclassical Theories and Applications. Advanced Structured Materials, Vol. 11 - Berlin: Springer-Verlag, 2011

#### Co-author of English textbooks/monographs

- Altenbach, H.; Altenbach, J.; Kissing, W.: Bending of Beams and Plates Composed of Composite Materials - An Introduction into the Classical and Advanced Theories. Lublin: Lubielskie Towarzystwo Naukowe, 2001
- Altenbach, H.; Altenbach, J.; Kissing, W.: Mechanics of Composite Structural Elements. Berlin: Springer-Verlag, 2004
- Naumenko, K.; Altenbach, H.: Modeling of Creep for Structural Analysis. Berlin: Springer-Verlag, 2007 (Series: Foundations of Engineering Mechanics)

### **Scientific interests**

- Continuum Mechanics
  - Creep Mechanics,
  - Damage Mechanics,
  - Binary Media,
  - Foams
- Structural Mechanics
  - Beams, Plates, Shells, Thin-walled Structures,
  - Laminates and Sandwiches,
  - Functionally Graded Materials,
  - Nanostructures
- Failure & Strength Theories

### **Grants & collaborations**

- 1997 2001 EU-Project together with TU Lublin "Restructurization of the Mechanical Engineering Education at the TU Lublin
- 2005 2009 EU-Project (Transfer of Knowledge) with TU Lublin "Modern Composite Materials Applied in Aerospace, Civil and Mechanical Engineering: Theoretical Modelling and Experimental Verification"
- 2010 2013 EU-Project with TU Lublin "Center of Excellence for modern composites applied in aerospace and surface transport infrastructure"
- 2004 2005 EU-Project (Coordinator) on Education in the Ukraine with respect to the Bologna Statement
- 1992 2002, 2002 -2009, 2010 -2014 Member of Graduate Schools at the University of Magdeburg
- Siemens, BorgWarner

http://tm.iw.uni-halle.de/

http://www.uni-magdeburg.de/ifme/ifme.html

holm.altenbach@ovgu.de

## **Continuum Mechanics**

### Lectures on the History and Basics

#### Holm Altenbach

Otto-von-Guericke University Magdeburg, Germany

Institute of Mathematics Sibirian Federal University Krasnoyarsk, Russian Federation

#### Before we start ...

#### **Preliminary Reading**

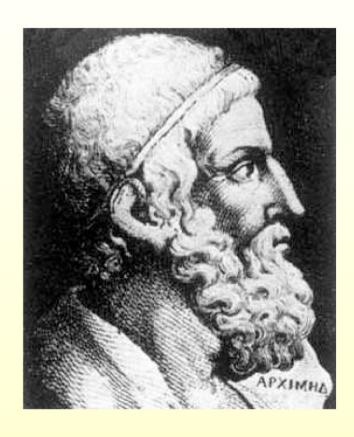
- Lemaitre, J., Chaboche, J.-L. Mechanics of Solid Materials, 1994,
   Cambridge University Press
- Lurie, A.I. Nonlinear Theory of Elasticity, 1990, North-Holland
- Palmov, V. Vibrations of Elasto-Plastic Bodies, 1998, Springer
- Haupt, P. Continuum Mechanics and Theory of Materials, 2002, Springer
- Lurie, A.I. Theory of Elasticity, 2005, Springer
- Rubin, D., Lai, W.M., Krempl, E. Introduction to Continuum Mechanics, 2009, Elsevier
- Lebedev, L., Cloud, M., Eremeyev, V. Tensor Analysis with Applications to Mechanics, 2010, World Scientific Publishing Company



### **Historical Remarks**

## **Early Period**

## First Steps - Archimedes of Syracuse

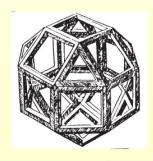


- Born c. 287 BC, Syracuse, Sicily
- Died c. 212 BC, Syracuse
- Residence Syracuse, Sicily
- Fields Mathematics, Physics, Engineering, Astronomy, Invention
- Known for Archimedes'
   Principle, Archimedes' screw,
   Hydrostatics, Levers,
   Infinitesimals

### Lever

- The earliest remaining writings regarding levers date from the 3rd century BC and were provided by Archimedes Give me a place to stand, and I shall move the earth with a lever.
  - This is a remark of Archimedes who formally stated the correct mathematical principle of levers (quoted by Pappus of Alexandria).
- It is assumed that in ancient Egypt, constructors used the lever to move and uplift obelisks weighting more than 100 tons.
- The force applied (at end points of the lever) is proportional to the ratio of the length of the lever arm measured between the fulcrum (pivoting point) and application point of the force applied at each end of the lever. Mathematically, this is expressed by M=Fd.







Renaissance - Industrial Revolution

#### **Historical Remarks**

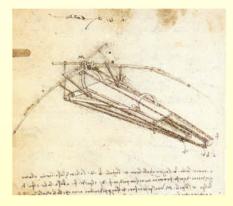
**Renaissance - Industrial Revolution** 

## Leonardo da Vinci

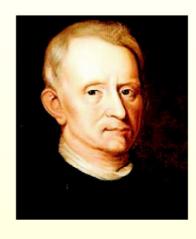


Holm Altenbach (OVG)

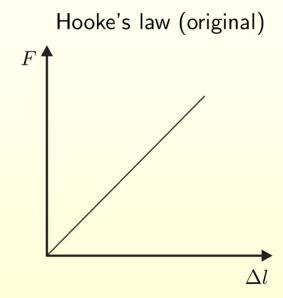
- Born April 15, 1452, Vinci, Florence
- Died May 2, 1519 Amboise, Touraine
- Nationality Italian
- Fields Many and diverse fields of arts and sciences
- Known for Flying machine



## Hooke's<sup>1</sup> Contributions

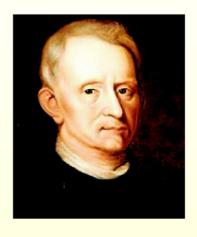


- ceiiinossssttuv (1676)
- Ut tensio, sic vis (1678)
- As the extension, so the force.

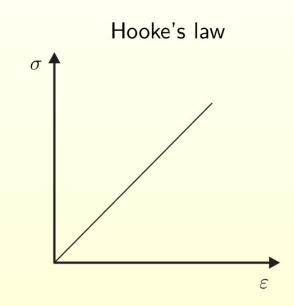


<sup>&</sup>lt;sup>1</sup>Robert Hooke (18 July 1635 – 3 March 1703)

# Hooke's<sup>1</sup> Contributions



- ceiiinossssttuv (1676)
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<sup>&</sup>lt;sup>1</sup>Robert Hooke (18 July 1635 – 3 March 1703)

## **Analogy - Elastic Spring**



For systems that obey

Hooke's law,

the extension produced
is directly proportional

to the load

 $| \boldsymbol{F} = -k \boldsymbol{x} |$ 

Modern form of the Hooke's law

$$\sigma = E \varepsilon$$

 $\sigma$  stress,  $\varepsilon$  strain, E Young's modulus

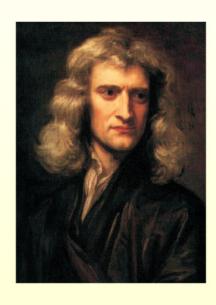
## Bernoulli's<sup>2</sup> Contributions



#### 1686

postulate of the angular momentum

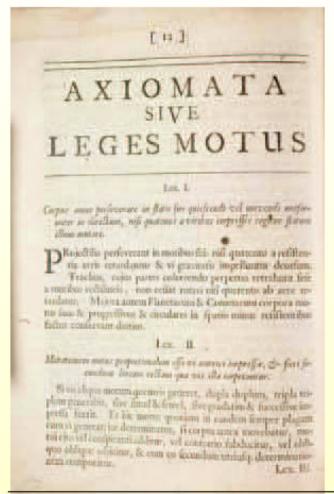
## **Newton's<sup>3</sup> Contributions**



#### **Newton's Axioms**

- $\frac{d}{dt}(m\boldsymbol{v}) = \boldsymbol{0}$  (statics)
    $\frac{d}{dt}(m\boldsymbol{v}) = \boldsymbol{F}$  (kinetics)
- actio = reactio
- superposition

## Newton's Laws<sup>4</sup>



<sup>&</sup>lt;sup>4</sup>Newton's First and Second laws, in Latin, from the original 1687 edition of

# Newton's Laws (Con'd)

#### Newton's first law: law of inertia

There exists a set of inertial reference frames relative to which all particles with no net force acting on them will move without change in their velocity. This law is often simplified as

A body persists its state of rest or of uniform motion unless acted upon by an external unbalanced force.

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Newton's first law is often referred to as the law of inertia.

# Newton's Laws (Con'd)

#### Newton's second law: law of motion

Observed from an inertial reference frame, the net force on a particle is proportional to the time rate of change of its linear momentum:

 $\boldsymbol{F} = d(m\boldsymbol{v})/dt$ . This law is often stated as

Force equals mass times acceleration (F = ma).

The net force on an object is equal to the mass of the object multiplied by its acceleration.

#### Newton's third law: law of reciprocal actions

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction. The strong form of the law further postulates that these two forces act along the same line. This law is often simplified into the sentence

To every action there is an equal and opposite reaction.

# Newton's Laws (Con'd)

#### Newton's fourth law: law of superposition

Forces acting on a mass point or rigid body can be added as vectors.

#### **Problems**

- The axioms are valid for point bodies in the given form.
- How formulate the theory for rigid bodies?
- What can we do if the body is deformable?

## **Euler's<sup>5</sup> Contributions<sup>6</sup>**



#### **Euler's laws of motion**

- Balance of momentum
- Balance of moment of momentum

#### **Assumptions**

- Independence of translations and rotations
- Force and moment actions

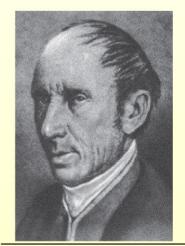
<sup>&</sup>lt;sup>5</sup>Leonhard Euler (15 April 1707 - 18 September 1783)

<sup>&</sup>lt;sup>6</sup>C. Truesdell *Die Entwicklung des Drallsatzes* ZAMM **44** (1964) 4/5, 149–158

# Cauchy's <sup>7</sup> Contributions

#### **Definition of Stress**

Stress  $\sigma$  is a measure of the average amount of force F exerted per unit area A. It is a measure of the intensity of the total internal forces acting within a body across imaginary internal surfaces, as a reaction to external applied forces and body forces. It was introduced into the theory of elasticity by Cauchy around 1822.



$$\sigma = \frac{F}{A}$$

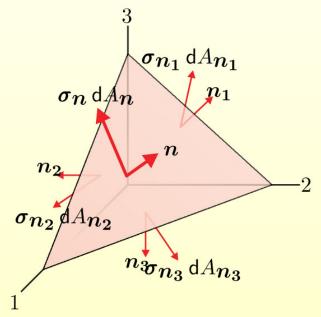
<sup>&</sup>lt;sup>7</sup>Augustin Louis Cauchy (21 August 1789 - 23 May 1857)

# **Cauchy's Contributions**

## Cauchy's Lemma

$$oldsymbol{\sigma_n} = n \cdot oldsymbol{\sigma}$$

Relation between the stress vector  $oldsymbol{\sigma}_n$  on the surface with the normal n and the stress tensor  $oldsymbol{\sigma}$ 



# Young's Modulus (I)

#### **Definition**

An elastic modulus, or modulus of elasticity, is the mathematical description of an object or substance's tendency to be deformed elastically (i.e., non-permanently) when a force is applied to it. The elastic modulus of an object is defined as the slope of its stress-strain curve in the elastic deformation region:

$$E \equiv \frac{\mathsf{stress}}{\mathsf{strain}}$$

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# Young's Modulus (II)

- Young's modulus (E) describes tensile elasticity, or the tendency of an object to deform along an axis when opposing forces are applied along that axis; it is defined as the ratio of tensile stress to tensile strain. It is often referred to simply as the elastic modulus.
- The shear modulus or modulus of rigidity  $(G \text{ or } \mu)$  describes an object's tendency to shear (the deformation of shape at constant volume) when acted upon by opposing forces; it is defined as shear stress over shear strain. The shear modulus is part of the derivation of viscosity.
- The **bulk modulus** (*K*) describes volumetric elasticity, or the tendency of an object's volume to deform when under pressure; it is defined as volumetric stress over volumetric strain, and is the inverse of compressibility. The bulk modulus is an extension of Young's modulus to three dimensions.

# **Thomas Young**<sup>8</sup>



#### Remark

The concept was developed in 1727 by **Leonhard Euler** and the first experiments that used the concept of Young's modulus in its current form were performed by the Italian scientist **Giordano Riccati** in 1782 - predating Young's work by 25 years.

<sup>8</sup>13 June 1773 - 10 May 1829

## Poisson's Contributions



### **Isotropic Linear Elastic Behavior (3D)**

two or one material parameters

$$\boldsymbol{\sigma} = \alpha \operatorname{tr}(\boldsymbol{\varepsilon}) \boldsymbol{I} + \beta \boldsymbol{\varepsilon} + \gamma \boldsymbol{\varepsilon}^{\mathsf{T}}$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^{\mathsf{T}} \Rightarrow \boldsymbol{\sigma} = K \operatorname{tr}(\boldsymbol{\varepsilon}) \boldsymbol{I} + 2\mu \boldsymbol{\varepsilon}$$

$$K$$
 bulk modulus,  $\mu$  shear modulus 
$$K=\frac{E}{3(1-2\nu)}, \quad \mu=G=\frac{E}{2(1+\nu)}$$

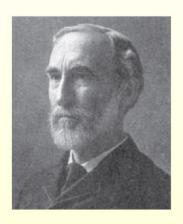
 $\nu$  - parameter or fixed?

Historical Remarks Late 19<sup>th</sup> - 20<sup>th</sup> Century

## **Historical Remarks**

Late 19<sup>th</sup> - 20<sup>th</sup> Century

## **Gibbs'**<sup>10</sup> Contributions



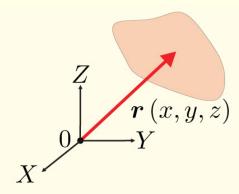
#### Tensor calculus

- index-free representation
- index representation

# Gibbs' Contributions (Con'd)

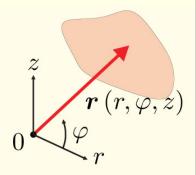
### Index-free representation

#### Cartesian coordinates



$$\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$$

### Cylindrical coordinates



$$r = re_r + r\varphi e_\varphi + ze_z$$

## Invariant representation

length: 
$$r = \sqrt{r \cdot r} = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + r^2 \varphi^2 + z^2}$$

# Cosserat<sup>11,12</sup> Brothers



### Cosserat Model (1896, 1909)

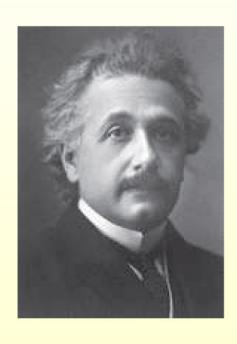
- extended continuum model
- balance of forces
- balance of angular momentum
- no constitutive equations

Eugéne Cosserat

<sup>&</sup>lt;sup>11</sup>François Cosserat (26 November 1852 - 22 March 1914)

<sup>&</sup>lt;sup>12</sup>Eugéne Cosserat (4 March 1866 - 31 May 1931)

# Einstein's<sup>13</sup> Contributions



#### Contributions to Mechanics

- Tensor Calculus
- Summation Convention
- Relativistic Mechanics

## Hilbert's<sup>14</sup> Problems



## 6<sup>th</sup> Problem (1900)

- Axiomatic Formulation of Mechanics
- unsolved up to now

# Lagally's<sup>15</sup> Contributions



#### Tensor Calculus

Invariant Representation

# Noll's<sup>16</sup> and Truesdell's<sup>17</sup> Contributions

#### Contributions to Mathematics and Mechanics

- Rational Mechanics
- Theory of Simple Materials
- Rational Thermodynamics
- History of Mechanics

<sup>&</sup>lt;sup>16</sup>Walter Noll (7 January 1925)

<sup>&</sup>lt;sup>17</sup>Clifford Abmbrose Truesdell III (18 February 1919 - 14 January 2000)

## **Tensor Calculus**

Tensor Calculus

Introduction

## **Tensor Calculus**

### Introduction

### Motivation

## **Two Notations**

The tensor calculus is a powerful tool for the description of the fundamentals in continuum mechanics and the derivation of the governing equations for applied problems.

Two possibilities for the representation of the tensors and the tensorial equations:

September 12<sup>th</sup> - 15<sup>th</sup>, 2011

## **Two Notations**

The tensor calculus is a powerful tool for the description of the fundamentals in continuum mechanics and the derivation of the governing equations for applied problems.

Two possibilities for the representation of the tensors and the tensorial equations:

the direct (symbolic, coordinate-free) notation and

## **Two Notations**

The tensor calculus is a powerful tool for the description of the fundamentals in continuum mechanics and the derivation of the governing equations for applied problems.

Two possibilities for the representation of the tensors and the tensorial equations:

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- the direct (symbolic, coordinate-free) notation and
- the index (component) notation

## **Direct Notation**

The direct notation operates with scalars, vectors and tensors as physical objects defined in the multi-dimensional space.

#### Here we are limit ourselves to the three-dimensional case.

- A vector (first rank tensor) a is considered as a directed line segment rather than a triple of numbers (coordinates).
- A second rank tensor A is any finite sum of ordered vector pairs  $A = a \otimes b + \ldots + c \otimes d$ .
- The direct notation is coordinate free and does not need an introduction of any preferred coordinate system.
- The scalars, vectors and tensors are handled as invariant (independent from the choice of the coordinate system) quantities.

Continuum Mechanics

## **Some Basic References**

The advantages of the direct notation is the reason for the use of the direct notation in the modern literature of mechanics and rheology, e.g.

- H. Giesekus: Phänomenologische Rheologie, Springer, 1994
- J. & H. Altenbach: Einführung in die Kontinuumsmechanik, Teubner, 1994
- S. Antman: Nonlinear Problems of Elasticity, Springer, 1995
- V. Palmov: Vibrations in Elasto-Plastic Bodies, Springer, 1998
- P.A. Zhilin: Vectors and second rank tensors in three-dimensional space (in Russ.), Nestor, 2001
- P. Haupt: Continuum Mechanics and Theory of Materials, Springer, 2002

Continuum Mechanics

A.I Lurie: Linear Theory of Elasticity, Springer, 2005

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### **Index Notation**

The index notation deals with components or coordinates of vectors and tensors. For a selected basis, e.g.  $\boldsymbol{g}_i$ , i=1,2,3 one can write

$$oldsymbol{a} = a^i oldsymbol{g}_i, \quad oldsymbol{A} = \left(a^i b^j + \ldots + c^i d^j\right) oldsymbol{g}_i \otimes oldsymbol{g}_j$$

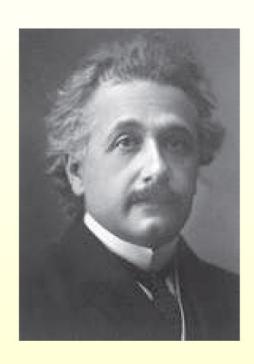
Here the **Einstein's summation convention** is used: in one expression the twice repeated indices are summed up from 1 to 3

$$a^k \boldsymbol{g}_k \equiv \sum_{k=1}^3 a^k \boldsymbol{g}_k, \quad A^{ik} b_k \equiv \sum_{k=1}^3 A^{ik} b_k$$

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In the above examples k is a so-called dummy index.

# **Albert Einstein** (\*14.03.1879, †18.04.1955)<sup>18</sup>



Born: March 14, 1879 in Ulm, Germany Died: April 18, 1955 in Princeton, USA

theoretical physicist

theory of relativity: mass-energy equivalence,  $E=mc^2$  1921 Nobel Prize in Physics: photoelectric effect

## **Index Notation - Basic Operations**

Within the index notation the basic operations with tensors are defined with respect to their coordinates, e. g. the sum of two vectors is computed as the sum of their coordinates

$$c^i = a^i + b^i$$

The introduced basis remains in the background. It must be noted that a change of the coordinate system leads to the change of the components of tensors.

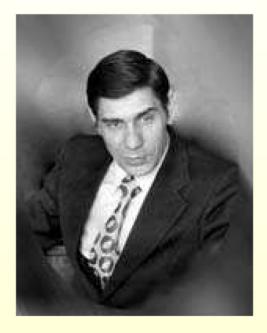
## **Some Additional References**

When solving applied problems the tensor equations can be "translated into the language" of matrices for a specified coordinate system. The purpose of these **Lectures** is to give a brief guide to notations and rules of the tensor calculus.

The calculus of matrices is presented in

- D.K. Faddejew, W.N. Faddejewa: Numerische Methoden der linearen Algebra, Deutscher Verlag der Wissenschaften, 1964
- R. Bellmann: Introduction to Matrix Analysis, McGraw Hill, 1970
- R. Zurmühl, S. Falk: Matrizen und ihre Anwendungen, Springer, 1992

# **Pavel Andreevich Zhilin** (\*08.02.1942, †04.12.2005)



Russian (Soviet) Mechanist Professor of Rational Mechanics Polytechnical University St. Petersburg

ZAMM (Z. Angew. Math. Mech.) 87, No. 2, 79 - 80 (2007)

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